LBNPS JAUNPUR CLASS X

Subject- Math

CHAPTER- 1 (REAL NUMBERS)

Natural Numbers – Counting numbers 1, 2, 3, ... are known as natural numbers.

Whole Numbers – All counting numbers together with 0 form the collection of whole numbers

Ex- 0, 1, 2, 3, 4, 5, ... etc.

Integers – All counting numbers, negatives of counting numbers and 0 form the collection of all integers.

Thus - .... -4, -3, -2, -1, 0, 1, 2, 3 ..... etc.

Rational Numbers – The numbers of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are called rational numbers.

Rational Numbers in Decimal Form –

(1) Terminating Rational Decimal –

If $\frac{p}{q}$ is a rational number where $q \neq 0$ then, if q is a factor of $2^m \times 5^n$

$$\frac{p}{q} = \text{terminating decimal} = \frac{p}{2^m \times 5^n}$$

(2) Non-Terminating Decimal –

$\frac{p}{q}$ will be non-terminating decimal if $q \neq 0$ and q is not a factor of $2^m \times 5^n$.

So, if $q \neq 2^m \times 5^n$

Then $\frac{p}{q}$ will non-terminating decimal.

Note :- m & n are ~~are~~ some non-negative integers.

Ex- $\frac{33}{55} = \frac{33}{11 \times 5}$ ~~not~~ non-terminating decimal

$\frac{41}{1000} = \frac{41}{10^3} = \frac{41}{(2 \times 5)^3}$ = terminating decimal.

Ex- Express the given rational number in simplest form.

(i) $0.\overline{6}$ (ii) $1.\overline{8}$

Solution – (i) Let $x = 0.\overline{6}$

Then $x = 0.666 \ldots$ ——— (1)

$\therefore$ $10x = 6.666 \ldots$ ——— (2)

eqn (2) – (1) (on substracting)

$9x = 6$

$x = 6/9 = 2/3$ Ans.

(ii) $x = 1.\overline{8}$

$\Rightarrow$ $x = 1.888 \ldots$ (i)

$10x = 18.888 \ldots$ (ii)

On substracting (i) from (ii)

$9x = 17$

$x = 17/9 = 1\frac{8}{9}$ Ans.

Irrational Numbers –

The numbers which when ~~not~~ expressed in decimal form are expressible as nonterminating and nonrepeating decimals are know as irrational numbers.

Type 1. "— Every nonterminating and nonrepeating decimal is irrational."

Type 2. If m is a positive integer which is not a perfect square then $\sqrt{m}$ is irratinal.

Ex- $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{10} \ldots$ etc.

Type 3. If m is a positive integer which is not a perfect cube then $\sqrt[3]{m}$ is irrational.

Ex- $\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5} \ldots$ etc.

Type 4. π is irrational, while $\frac{22}{7}$ is rational.

Theorem 1. Let p be a prime number and 'a' be a positive integer. If p divides a^2 then show that p divides a.

Let 'p' be a prime number and 'a' be a positive integer such that p divides a^2.

We know that

positive integer = product of primes

Let $a = P_1 P_2 \cdots P_n$

Then $a^2 = P_1^2 P_2^2 \cdots P_n^2$

Now, p divides a^2.

$\Rightarrow$ p is a prime factor of a^2.

$\Rightarrow$ p is one of $P_1, P_2, P_3, \cdots P_n$

$\because$ Prime factors of a^2 are $P_1, P_2, \cdots P_n$

$\Rightarrow$ p divides a.

Thus, p divides a^2

$\Rightarrow$ p divides a.

Theorem 2. Prove that $\sqrt{2}$ is irrational.

let $\sqrt{2}$ be rational and its simplest form is $\frac{p}{q}$, where $q \neq 0$

Now, $\sqrt{2} = \frac{p}{q}$ $\Rightarrow 2 = \frac{p^2}{q^2}$

$\Rightarrow 2q^2 = p^2$ ——— (i)

$\Rightarrow$ 2 divides p^2 [$\because$ 2 divides $2q^2$]

$\Rightarrow$ 2 divide p

[$\because$ 2 is prime and divides $q^2 \Rightarrow 2$ divides q]

Let $p = 2c$ for some integer c.

Putting $p = 2c$ in eq (i)

$$2q^2 = 4c^2 \Rightarrow q^2 = 2c^2$$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ 2 divides q.

Thus 2 is common factor of p and q.

But, this contradicts the fact that p and q have no common factor other than 1.

The contradict arises by assuming that $\sqrt{2}$ is rational

Hence, $\sqrt{2}$ is rational.

Que. Prove that $\sqrt{3}$ is irrational.

Que. Prove that $\sqrt{11}$ is irrational.

Questions

Que 1. Prove that $\sqrt{5}$ is irrational.

Que 2. Prove that $3+2\sqrt{5}$ is irrational.

Que 3. State whether the following rational numbers will have a terminating or non-terminating decimal expansions.

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{129}{2^2 5^7 7^5}$ (iv) $\frac{6}{15}$

Que 4. The following real numbers have decimal expansions as given below. In each case whether they are rational or not. If they are rational and of the form $\frac{p}{q}$, what you say about prime factors of q.

(i) 43.123456789

(ii) 0.120120012000120000----

(iii) $43.\overline{123456789}$

Que 5. Write the simplest form of given rational numbers.

(i) 0.375 (ii) 0.0875 (iii) 23.3408