LORD BUDDHA NATIONAL PUBLIC SCHOOL, JAUNPUR
CLASS – IX
SUBJECT – MATHS

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CHAPTER – NUMBER SYSTEM (PART-II)

Terminating and Recurring Decimals

Terminating Decimals → Every fraction $\frac{p}{q}$ can be expressed as a decimal.
If the decimal expression of p/q terminates i.e., comes to an end, then the decimal so obtained is called a terminating decimal.

E.g. → $\frac{1}{4} = 0.25$, $\frac{5}{8} = 0.625$, $2\frac{2}{5} = \frac{13}{5} = 2.6$

Thus, each of the numbers $\frac{1}{4}$, $\frac{5}{8}$ & $2\frac{2}{5}$ can be expressed in the form of a terminating decimal.

NOTE A fraction p/q is a terminating decimal only when prime factors of q are 2 and 5 Only.

E.g → Each one of the fractions $\frac{1}{2}, \frac{3}{4}, \frac{7}{20}, \frac{13}{25}$ is a terminating decimal, since the denominator of each has no prime factor other than 2 and 5.

Example Without actual division, find which of the following rational numbers are terminating decimals:

(i) $\frac{5}{32}$ ⇒ Denominator of $\frac{5}{32}$ is 32

& $32 = 2^5$

∴ 32 has no prime factor other than 2.

So, $\frac{5}{32}$ is terminating decimal.

(ii) $\frac{11}{24} \Rightarrow$ Denominator of $\frac{11}{24}$ is 24.

And, $24 = 2^3 \times 3$.
Thus 24 has a prime factor 3, which is other than 2 or 5.
$\therefore \frac{11}{24}$ is not a terminating decimal.

(iii) $\frac{27}{80} \Rightarrow$ Denominator of $\frac{27}{80}$ is 80.

And $80 = 2^4 \times 5$
Thus, 80 has no prime factors other than 2 and 5.
$\therefore \frac{27}{80}$ is a terminating decimal.

Repeating (or Recurring) Decimals

A decimal in which a digit or a set of digits repeats periodically, is called a repeating or recurring decimal.

E.g. → $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$

$\frac{3}{11} = 0.272727\ldots = 0.\overline{27}$

$\frac{15}{7} = 2.142857142857\ldots = 2.\overline{142857}$

$\frac{11}{6} = 1.8333\ldots = 1.8\overline{3}$

In recurring decimal, we place a bar over the first block of the repeating part and omit the other repeating blocks.

Special Characteristics of Rational Numbers

Every rational number is expressible either as a terminating decimal or as a repeating decimal.

Every terminating decimal is a rational number.

Every repeating decimal is a rational number.

Examples

Eg (1) Express $3\frac{1}{8}$ in decimal form.

⇒ We have, $3\frac{1}{8} = \frac{25}{8}$

By actual division, we have

$$
\begin{array}{r}
3.125 \\
8 \overline{)25.000} \\
-24 \\
\hline
10 \\
-8 \\
\hline
20 \\
-16 \\
\hline
40 \\
-40 \\
\hline
\times
\end{array}
$$

$\therefore \frac{25}{8} = 3.125.$

Eg (2) Express $\frac{2}{11}$ in decimal form

⇒ By actual division, we have

$$
\begin{array}{r}
0.1818\ldots \\
11 \overline{)2.0} \\
-0 \\
\hline
20 \\
-11 \\
\hline
90 \\
-88 \\
\hline
20 \\
-11 \\
\hline
90 \\
-88 \\
\hline
2
\end{array}
$$

$\therefore \frac{2}{11} = 0.1818\ldots$

$= 0.\overline{18}$.

E.g. → Express the following decimals as a fraction in simplest form.

(a) $0.\overline{3}$

⇒ Let $x = 0.\overline{3}$

then, $x = 0.3333...$ ——— ①

$\therefore 10x = 3.333...$ ——— ②

On subtracting ① from ②, we get

$10x - x = 3.333... - 0.333...$

$9x = 3$

$x = \frac{\cancel{3}}{\cancel{9}}$

$x = \frac{1}{3}$

⇒ Hence $0.\overline{3} = \frac{1}{3}$

(b) $1.\overline{4}$

⇒ Let $x = 1.\overline{4}$

then $x = 1.444...$ —①

$\therefore 10x = 14.444...$ ②

On subtracting ① from ②, we get.

$10x - x = 14.444... - 1.444...$

$9x = 13$

$x = \frac{13}{9}$

⇒ Hence $1.\overline{4} = \frac{13}{9}$.

(c) $0.\overline{36}$

⇒ Let $x = 0.\overline{36}$

then, $x = 0.363636...$ —①

Here the bar is on two digits together,

so we will multiply eqⁿ ① by 100, we get

$100x = 36.3636...$ —— ②

On subtracting ① from ②, we get.

$100x - x = 36.363636 - 0.363636...$

$99x = 36$

$x = \frac{36}{99} = \frac{4}{11}$

Hence $0.\overline{36} = \frac{4}{11}$.

(d) $0.5\overline{7}$

Let $x = 0.5\overline{7}$

then $x = 0.5777$ ---- ①

Here the bar starts after one decimal place. Hence we will multiply eqⁿ ① by 10 first then by 10 again since the bar is on a single digit. We get,

$10x = 5.777$ --- ②

$100x = 57.777$ --- ③

On subtracting eqⁿ ② from ③, we get

$100x - 10x = 57.777... - 5.777...$

$90x = 52$

$x = \frac{52}{90}$

$x = \frac{26}{45}$

Hence $0.5\overline{7} = \frac{26}{45}$.

EXERCISE

① Without actual division, find which of the following rationals are terminating decimal

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(i) 13/80 (ii) 7/24 (iii) 8/35
(iv) 5/12 (v) 16/125 (vi) 4/30
(vii) 7/120 (viii) 3/1000 (ix) 1/180
(x) $\frac{5}{70}$

(2). Convert each of the following into a decimal and classify them as terminating, non terminating or repeating.

(i) 5/8 (ii) $2\,5/12$ (iii) $5\frac{1}{7}$
(iv) 9/16 (v) 3/7 (vi) 4/11
(vii) 7/25 (viii) $3\frac{1}{3}$ (ix) 26/90
(x) 11/24

(3) Express each of the following as a fraction in simplest form.

(i) $0.\overline{3}$ (ii) $0.\overline{34}$ (iii) $3.\overline{14}$
(iv) $1.\overline{3}$ (v) $0.\overline{324}$ (vi) $0.\overline{17}$
(vii) $1.1\overline{23}$ (viii) $0.5\overline{4}$ (ix) $0.1\overline{63}$
(x) $0.2\overline{45}$ (xi) $0.\overline{001}$ (xii) $0.\overline{89}$
(xiii) $0.3\overline{6}$ (xiv) $0.28\overline{4}$ (xv) $3.\overline{35}$
(xvi) $2.41\overline{6}$ (xvii) $3.41\overline{42}$ (xviii) $3.1\overline{6}$
(xix) $5.\overline{28}$ (xx) $6.\overline{17}$