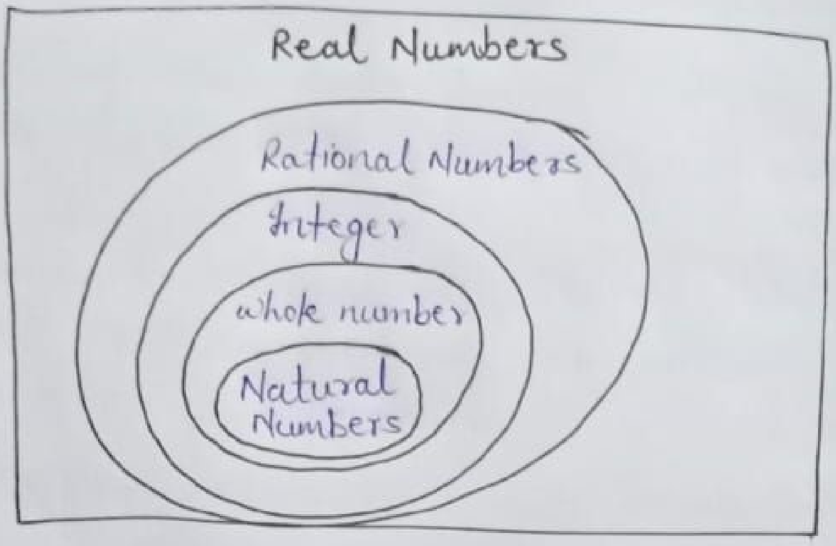
LBNPS Jaunpur

CLASS X

Subject - Math

Chapter 1   REAL NUMBERS

Real Numbers :- A real number is a value of a continuous quantity that can represent distance along a line.



- **Lemma** :- A lemma is a proven statement used for proving another statement.

- Euclid's Division Lemma :-

For any two given positive integers a and b, there exist unique whole numbers q and r such that

$$a = bq + r, \quad \text{where} \quad 0 \leq r < b$$

a = dividend
b = divisor
q = quotient
r = remainder

$\boxed{\text{Dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}}$

- **Algorithm** :- An algorithm is a series of well-defined steps which gives a method for solving a certain type of problem.

- **Euclid's Division Algorithm** :-

It is a technique to compute the HCF of two given positive integers, say a and b with $a > b$ - in the following steps —

Step 1. On dividing a by b, we get quotient q and remainder r such that

$$a = bq + r \quad , \text{ where } 0 \le r < b$$

Step 2. If $r = 0$ then $HCF(a, b) = b$

If $r \neq 0$ then apply the division lemma to b and r.

Step 3. Continue the process till the remainder is 0. The last divisor will be the required HCF.

For ex- Use euclid's algorithm find the HCF of 272 and 1032.

Let $a > b$

So, $a = 1032$ and $b = 272$

```
272) 1032 (3
     816
     ----
     216) 272 (1
          216
          ----
           56) 216 (3
               168
               ----
                48) 56 (1
                    48
                    ---
                     8) 48 (6
                        48
                        --
                         0
```

HCF = 8 **Ans.**

Applications of Euclid's Division Lemma :-

Que:- Show that every positive odd integer is of the form $(6m+1)$ or $(6m+3)$ or $(6m+5)$ for some integer m.

Solution Let n be a given positive odd integer

On dividing n by 6, let m be the quotient and r be the remainder.

Then By Euclid's division lemma, we have

$$n = 6m + r \quad , \text{ where, } \quad 0 \leq r < 6$$

$$\Rightarrow \quad n = 6m + r \quad , \text{ where } \quad r = 0, 1, 2, 3, 4, 5$$

$$n = 6m \text{ or } (6m+1) \text{ or } (6m+2) \text{ or } (6m+3) \text{ or } (6m+4) \text{ or } (6m+5)$$

But $n = 6m$, $(6m+2)$, $(6m+4)$ given even values of n.

Thus when n is odd, it is of the form $(6m+1)$ or $(6m+3)$ or $(6m+5)$ for some integer m.

Prime Factorisation Method to find HCF and LCM :-

Let a and b are the given numbers

Then, HCF of a and b is —

HCF (a, b) = Product of common terms with lowest power

LCM (a, b) = product of prime factors with highest power

Ex- Find the HCF and LCM of 126 and 156 using prime factorisation method.

$a = 126$
$b = 156$

common factor

$a = 126 = 2 \times 3 \times 3 \times 7 = 2^1 \times 3^2 \times 7$

$b = 156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3^1 \times 13$

$\therefore$ $HCF(126, 156) = 2^1 \times 3^1 = 6$ Ans.

and $LCM(126, 156) = 2^2 \times 3^2 \times 7 \times 13 = 4 \times 9 \times 7 \times 13 = 3276$

Ans.

Relation between HCF and LCM

$$\boxed{a \times b = HCF(a,b) \times LCM(a,b)}$$

Ex- Given that $HCF(252, 594) = 18$, find $LCM(252, 594)$.

We have

$$LCM_{(a,b)} \times HCF_{(a,b)} = a \times b$$

$$LCM(a,b) = \frac{a \times b}{HCF(a,b)}$$

$$LCM_{(252,594)} = \frac{252 \times 594}{18} = 8316$$

Ans.

Questions Based on Real Numbers.

Que ① Use EUCLID'S DIVISION ALGORITHM to find the HCF of –

ⓐ 135 and 225 ⓑ 196 and 38220 ⓒ 867 and 255

Que ② Use euclid's division lemma to show that the square of any positive integer is either of the form 3m or (3m+1) for some integer m.

Que ③ Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two no.

ⓐ 26 and 91 ⓑ 510 and 92

Que ④ Given that HCF(306, 657) = 9, find LCM(306, 657).

Que ⑤ The HCF of two numbers is 23 and their LCM is 1449. If one of the number is 161, find other.

Que ⑥ Is it possible to have two numbers whose HCF is 18 and LCM is 760 ? Give reason.

Que ⑦ Find the greatest number of four digits which is exactly divisible by 15, 24, 36.

Que ⑧ Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85cm and 12m 95cm.