LORD BUDDHA NATIONAL PUBLIC SCHOOL, JAUNPUR

CLASS – IX

SUBJECT – MATHS

PG. (1)

CHAPTER – NUMBER SYSTEM (PART-III)

Rationalisation

Suppose we are given a number whose denominator is irrational. Then, the process of converting its denominator to a rational number by multiplying its numerator and denominator by a suitable number, is called rationalisation.

E.g → Simplify $\frac{3}{\sqrt{5}}$ by rationalising the denominator.

Solution On multiplying the numerator and denominator of the given number by $\sqrt{5}$, we get

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}.$$

Rationalising factor (RF)

If the product of two irrational numbers is rational then each one is called the rationalising factor of the other.

If a and b are integers, x and y are natural numbers, then

(i) $(a+\sqrt{b})$ and $(a-\sqrt{b})$ are Rf of each other as $(a+\sqrt{b}) \times (a-\sqrt{b}) = (a^2-b)$ which is rational.

(ii) $(a+b\sqrt{x})$ and $(a-b\sqrt{x})$ are Rf of each other

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as $(a+b\sqrt{x})\times(a-b\sqrt{x}) = (a^2-b^2x)$, which is rational.

(iii) $(\sqrt{x}+\sqrt{y})$ and $(\sqrt{x}-\sqrt{y})$ are RF of each other.
as $(\sqrt{x}+\sqrt{y})\times(\sqrt{x}-\sqrt{y}) = (x-y)$ which is rational.

for rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

E.g. (1) Rationalise the denominator of $\frac{6}{\sqrt{5}+\sqrt{2}}$.

Solution $\frac{6}{\sqrt{5}+\sqrt{2}} = \frac{6}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{6(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{6(\sqrt{5}-\sqrt{2})}{5-2} = \frac{2\cancel{6}(\sqrt{5}-\sqrt{2})}{\cancel{3}}$

$= 2(\sqrt{5}-\sqrt{2})$

E.g (2) Rationalise the denominator of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Solution $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})\times(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{(\sqrt{3})^2+(\sqrt{2})^2+2\times\sqrt{3}\times\sqrt{2}}{3-2}$

$= \frac{3+2+2\sqrt{6}}{1} = (5+2\sqrt{6})$.

E.g (3) If a and b are rational numbers and $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a-b\sqrt{77}$. find a and b.

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Solution $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{(\sqrt{11}-\sqrt{7})}{(\sqrt{11}+\sqrt{7})} \times \frac{(\sqrt{11}-\sqrt{7})}{(\sqrt{11}-\sqrt{7})} = \frac{(\sqrt{11}-\sqrt{7})^2}{(\sqrt{11})^2-(\sqrt{7})^2}$

$= \frac{(\sqrt{11})^2+(\sqrt{7})^2-2\times\sqrt{11}\times\sqrt{7}}{11-7}$

$= \frac{11+7-2\sqrt{77}}{4} = \frac{18-2\sqrt{77}}{4}$

$= \frac{18}{4} - \frac{2}{4}\sqrt{77} = \frac{9}{2} - \frac{1}{2}\sqrt{77} = a - b\sqrt{77}$

$\Leftrightarrow a = \frac{9}{2}$ and $b = \frac{1}{2}$.

E.g. (4) If $x = (3+\sqrt{8})$. find the value of $x^2+\frac{1}{x^2}$

Solution Given : $x = 3+\sqrt{8}$

$\frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{1}{3+\sqrt{8}} \times \frac{(3-\sqrt{8})}{(3-\sqrt{8})} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}$

$= \frac{3-\sqrt{8}}{9-8} = \frac{3-\sqrt{8}}{1}$

$\therefore \left(x+\frac{1}{x}\right) = (3+\sqrt{8}) + (3-\sqrt{8}) = 6$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 6^2 = 36$

$\Rightarrow x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x} = 36$

$\Rightarrow x^2+\frac{1}{x^2}+2 = 36$

$\Rightarrow x^2+\frac{1}{x^2} = 36-2 = 34.$

Hence, $\left(x^2+\frac{1}{x^2}\right) = 34.$

E.g. (5) Simplify: $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$

Solution Taking L.C.M. of above no:

$\frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}) + (\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

$$\frac{(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{3})^2+(\sqrt{2})^2+2\times\sqrt{3}\times\sqrt{2}+(\sqrt{3})^2+(\sqrt{2})^2-2\times\sqrt{3}\times\sqrt{2}}{3-2}$$

$$= \frac{3+2+3+2}{1} = 10$$

Laws of Radicals

Let $a>0$ be a real number, and p and q be rational numbers, then we have

(i) $a^p \times a^q = a^{(p+q)}$ (ii) $(a^p)^q = a^{pq}$

(iii) $\frac{a^p}{a^q} = a^{(p-q)}$ (iv) $a^p \times b^p = (ab)^p$

Eg. → Simplify:

(a) $3^{3/4} \times 3^{1/4} = 3^{\frac{3}{4}+\frac{1}{4}} = 3^{\frac{4}{4}} = 3^1 = 3$
$[\because a^p \times a^q = a^{(p+q)}]$

(b) $\frac{5^{1/3}}{5^{1/8}} = 5^{1/3-1/8} = 5^{\frac{8-3}{24}} = 5^{5/24}$
$[\because a^p/a^q = a^{(p-q)}]$

(c) $3^{2/3} \times 7^{2/3} = (3\times7)^{2/3} = (21)^{2/3}$
$[\because a^p \times b^p = (ab)^p]$

(d) $(16)^{3/2} = (2^4)^{3/2} = 2^{4\times\frac{3}{2}} = 2^6 = 64$.

(e) $(125)^{-\frac{1}{3}} = \left(\frac{1}{125}\right)^{\frac{1}{3}} = \left(\frac{1}{5^3}\right)^{\frac{1}{3}} = \frac{1}{5^{3\times\frac{1}{3}}} = \frac{1}{5}$

(f) $(27)^{2/3} = (3^3)^{\frac{2}{3}} = 3^{3\times\frac{2}{3}} = 3^2$
$= 9$

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EXERCISE

(I) Rationalise the denominator of each of the following:

(1) $\frac{1}{\sqrt{7}}$ (2) $\sqrt{5}/2\sqrt{3}$ (3) $1/2+\sqrt{3}$

(4) $\frac{1}{\sqrt{5}-\sqrt{2}}$ (5) $\frac{1}{(5+3\sqrt{2})}$ (6) $\frac{1}{\sqrt{6}-\sqrt{5}}$

(7) $\frac{4}{(\sqrt{7}+\sqrt{3})}$ (8) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (9) $\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

(II) Find the value of a and b in each of the following:

(10) $\frac{\sqrt{3}+1}{\sqrt{3}-1} = a+b\sqrt{3}$ (11) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$

(12) $\frac{5-\sqrt{6}}{5+\sqrt{6}} = a-b\sqrt{6}$ (13) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-b\sqrt{3}$

(14) Simplify: $\left(\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1}\right)$

(15) Simplify: $\left(\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}\right)$

(16) If $x=(4-\sqrt{15})$. find the value of $x+\frac{1}{x}$

(17) If $x=(2+\sqrt{3})$, find the value of $x^2+\frac{1}{x^2}$

(18) Show that $\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{(\sqrt{6}-\sqrt{5})}$

$+\frac{1}{(\sqrt{5}-2)} = 5$

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(19) Simplify:

(a) $6^{2/5} \times 6^{3/5}$ (b) $3^{1/2} \times 3^{1/3}$

(c) $7^{5/6} \times 7^{2/3}$ (d) $\frac{6^{1/4}}{6^{1/5}}$

(e) $\frac{8^{1/2}}{8^{2/3}}$ (f) $\frac{5^{-6/7}}{5^{2/3}}$ (g) $3^{1/4} \times 5^{1/4}$

(h) $2^{5/8} \times 3^{5/8}$ (i) $6^{1/2} \times 7^{1/2}$

(j) $(3^4)^{1/4}$ (k) $(3^{1/3})^4$ (l) $\left(\frac{1}{3^4}\right)^{1/2}$

(20) Evaluate:

(a) $(49)^{1/2}$ (b) $(125)^{1/3}$

(c) $(64)^{1/6}$ (d) $(25)^{3/2}$

(e) $(32)^{2/5}$ (f) $(81)^{3/4}$

(g) $(64)^{-\frac{1}{2}}$ (h) $(8)^{-\frac{1}{3}}$

(i) $(81)^{-\frac{1}{4}}$