LBNPS JAUNPUR

CHAPTER - 2 CLASS - X

Mathematics

(Polynomials)

Polynomials – An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where $a_n \neq 0$ is called a polynomial in x of degree n.

Ex- $2x + 6$ is a polynomial in x of degree 1.

(i) **Linear Polynomial** – A polynomial of degree 1 is called linear polynomial.

Ex- $p(x) = ax + b$, where $a \neq 0$

(ii) **Quadratic Polynomial** – A polynomial of degree 2 is called quadratic polynomial.

Ex $p(x) = ax^2 + bx + c$, where $a \neq 0$

(iii) **Cubic Polynomial** – A polynomial of degree 3 is called cubic polynomial.

Ex- $p(x) = ax^3 + bx^2 + cx + d$, where, $a \neq 0$

(iv) **Biquadratic Polynomial** – A polynomial of degree 4 is called biquadratic polynomial.

Ex- $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

where, $a \neq 0$.

Value of a polynomial at a given point –

If $p(x)$ is a polynomial in x and if 'α' is any real number then the value obtained by putting $x = \alpha$, in $p(x)$ is called the value of $p(x)$ at $x = \alpha$.

Ex- Find the value of $p(x)$ at $x = 2$.

$$p(x) = 2x^2 + 3x - 1$$

Sol^n.

$$p(x) = 2x^2 + 3x - 1$$

By putting $x = 2$

$$p(2) = 2 \times (2)^2 + 3 \times 2 - 1$$

$$= 2 \times 4 + 6 - 1$$

$$= 8 + 6 - 1$$

$$= 13$$

Ans.

- **Zeros of a polynomial** – A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0$.

Ex-

$$p(x) = x^2 - 2x - 3$$

at $x = 3$

$$p(3) = 3^2 - 2 \times 3 - 3 = 0$$

So, $x = 3$ is the zero of given polynomial.

Relation between zeros and coefficient of a quadratic polynomial :-

Let $p(x) = ax^2+bx+c$, where $a \neq 0$

Then, α and, 'α' and 'β' be the zeros of quadratic polynomial.

then, $(x-\alpha)$ and $(x-\beta)$ are the factors of $p(x)$.

$$\text{Sum of zeros} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$$

$$\text{Product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

Ex- Find the zeros of the polynomial $2x^2+5x-12$ and verify the relationship between its zeros and coefficients.

$$f(x) = 2x^2+5x-12$$
$$= 2x^2+8x-3x-12$$
$$= 2x(x+4)-3(x+4)$$
$$f(x) = (x+4)(2x-3)$$

For zeros —

$$f(x) = 0 = (x+4)(2x-3)$$

$$\boxed{x = 3/2, -4}$$

Sum of zeros $(\alpha+\beta) = \frac{-b}{a} = -4+\frac{3}{2} = -\frac{5}{2}$

Product of zeros $(\alpha \cdot \beta) = -4 \times \frac{3}{2} = -\frac{12}{2}$

Ex- Find a quadratic polynomial, the sum and product of whose zeros are -5 and 6 respectively.

Solution - Let α and β be the zeros of the required polynomial $f(x)$.

Then, $\alpha + \beta = -5$

$\alpha \cdot \beta = 6$

$\therefore \quad f(x) = x^2 - (\alpha+\beta)x + \alpha \cdot \beta$

$= x^2 - (-5)x + 6$

$f(x) = x^2 + 5x + 6$ Ans.

Ex- If the product of zeros of the polynomial $(ax^2 - 6x - 6)$ is 4, find the value of a.

Solution - $\alpha \cdot \beta = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-6}{a}$

$\alpha \cdot \beta = 4$ is given -

So, $4 = -\dfrac{6}{a}$

$a = -\dfrac{6}{4} = -\dfrac{3}{2}$

$\boxed{a = -3/2}$ Ans

Questions (Level-1)

Que 1. Find the zeros of the following polynomials.

(i) $x^2+7x+12$ (ii) x^2-2x-8 (iii) $4x^2-4x-3$

Que 2. Find the quadratic polynomial whose zeros are 2 and -6. Verify the relation between the coefficient and the zeros of the polynomial.

Que 3. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence find the zeros of the polynomial.

Que 4. If $x = \frac{2}{3}$ and ~~$x=3$~~ $x=-3$ are the roots of the quadratic equation $ax^2+7x+b=0$ then find the value of a and b.

Que 5. If $(x+a)$ is a factor of the polynomial $2x^2+2ax+5x+10$, find the value of 'a'.

Que 6. Find the quadratic polynomial the sum of whose roots is $\sqrt{2}$ and their product is $1/3$.

Que 7. If one zero of the polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, find the value of 'a'.